WEST BENGAL STATE UNIVERSITY

B.Sc. Honours Part-III Examination, 2020

MATHEMATICS

PAPER-MTMA-VI

Time Allotted: 2 Hours Full Marks: 50

The figures in the margin indicate full marks.
Candidates should answer in their own words and adhere to the word limit as practicable.
All symbols are of usual significance.

GROUP-A

[Marks: 25]

Answer any *one* question from Question Nos. 1 to 3 and any *one* from Question Nos. 4 and 5

1. Answer any ***one*** question from the following: 5×1 = 5

(a) Give the classical and frequency definitions of probability. State the drawbacks of these definitions.

(b) Prove that the most probable number of successes in Bernoullian sequence of n trials is the integer(s) i_m given by the inequality

$$(n+1)p-1 \le i_m \le (n+1)p,$$

where p is the constant probability of success in each trial.

(c) Medical records show that two out of 19 persons in a certain town has a thyroid deficiency. If 15 persons in this town are randomly chosen and tested, what is the probability that at least one of them will have a thyroid deficiency? Also find the probability of exactly two persons having thyroid deficiency.

(d) A missile was fired at a plane on which there are two targets, T_1 and T_2. The probability of hitting T_1 is p_1 and that of hitting T_2 is p_2. It is known that T_2 was not hit. Find the probability that T_1 was hit.

(e) If $p = \frac{\mu}{n}$, where μ is a positive constant and $0 < p < 1$, n is a positive integer, then prove that $\lim\limits_{n\to\infty} \binom{n}{i} p^i (1-p)^{n-i} = e^{-\mu} \frac{\mu^i}{i\,!}$.

2. Answer any ***one*** question from the following: 5×1 = 5

(a) Let X be a continuous random variable and let $f_X(x)$ be the corresponding probability density function. Also let $y = g(x)$ be a continuously differentiable function for all values of x. If $f_Y(y)$ be the probability density function of the random variable Y, given by $Y = g(X)$ and if $\frac{dy}{dx}$ is either positive or negative for all x, then prove that $f_Y(y) = f_X(x)\left|\frac{dx}{dy}\right|$, where $y \in$ range of g.

(b) If X is a $\gamma(l)$ variate, find the probability density function of $\sqrt{X}$.

(c) Consider the random experiment of throwing a pair of dice. Let X denote the number of sixes and Y denote the number of fives that turn up. Find the joint p.m.f. of the two-dimensional random variable (X, Y) and the marginal p.m.f. of X and Y. Find the probability $P(X+Y \geq 2)$.

(d) State Tchebycheff's theorem and hence prove Bernoulli's limit theorem.

(e) Let X, Y be independent variates each having the density function $ae^{-ax}(0 < x < \infty)$, where a is a positive constant. Find the density function of $\frac{X}{Y}$. Prove that the variate $\frac{Y}{X+Y}$ is uniformly distributed over (0, 1).

3. Answer any ***one*** question from the following: 5×1 = 5

(a) Prove that the expectation $E(X)$ of a continuous random variable X, if it exists, can be expressed as $E(X) = \int_0^\infty \{1 - F(x) - F(-x)\}\,dx$ where $F(x)$ is the distribution function of X. 5

(b) Find the median of binomial $\left(5, \frac{1}{2}\right)$ distribution. 5

(c) Define concept of convergence in probability. Let $X_n \underset{\text{in } p}{\longrightarrow} a$ as $n \to \infty$ and $Y_n \underset{\text{in } p}{\longrightarrow} b$ as $n \to \infty$, then show that $X_n Y_n \underset{\text{in } p}{\longrightarrow} ab$ as $n \to \infty$. 5

(d) Find the mean and standard deviation of the continuous distribution with probability density function given by 5

$$f(x, y) = \begin{cases} 1 - |1 - x| & \text{if } 0 < x < 2 \\ 0 & \text{elsewhere} \end{cases}$$

(e) If X_1, X_2, X_3 be pairwise uncorrelated random variables, each having the same standard deviation, then find the correlation coefficient between $X_1 + X_2$ and $X_2 + X_3$. 5

4. (a) Define Sampling distribution and distribution of sample. What is standard error? Find the standard error of sample mean. 6

(b) Prove that the maximum likelihood estimate of the parameter α of the population having density function $f(x) = \frac{2(\alpha - x)}{\alpha^2}$, $(0 < x < \alpha)$ for a sample x_1 of unit size is $2x_1$ and that this estimate is biased. 7

(c) A random sample of size 10 was drawn from a normal population with an unknown mean and a variance of 44.1. If the observations are 65, 71, 80, 76, 78, 82, 68, 72, 65, 81, obtain a 95% confidence interval for the population mean. Given $\frac{1}{\sqrt{2\pi}} \int_{1.96}^{\infty} e^{-x^2/2}\,dx = 0.025$. 7

5. (a) Use the principle of least square to find the regression lines for the bivariate sample $(x_1, y_1), (x_2, y_2), ..., (x_n, y_n)$. Also measure the goodness of fit of the regression lines. 7

(b) Apply Neyman Pearson theorem to construct a test of null hypothesis $H_0 : m = m_0$ against an alternative $H_1 : m = m_1$ for a normal (m, σ) population, where σ is known and m_0, m_1 are two unequal numbers. 6

(c) Find by the method of likelihood ratio testing a test for the null hypothesis $H_0 : m = m_0$ for a normal (m, σ) population when σ unknown. 7

GROUP-B

SECTION-I

[Marks: 15]

Answer any *one* question from the following 15×1 = 15

6. (a) What do you mean by "round off" errors in numerical data? Show how these errors are propagated in a difference table. What is noise level? 7.5

(b) Show that the remainder in approximating $f(x)$ by the interpolation polynomial using distinct interpolating points $x_0, x_1, ..., x_n$ is of the form 7.5

$$(x-x_0)(x-x_1).....(x-x_n)\frac{f^{(n+1)}(\xi)}{(n+1)!},$$

where $\min\{x_0, ..., x_n\} < \xi < \max\{x_0, ..., x_n\}$.

7. (a) Prove the following for divided differences: 5+2.5

(i) $f(x_0, x_1, ..., x_n) = \dfrac{\Delta^n f(x_0)}{n!\, h^n}$, for equidistant arguments, where $x_r = x_0 + rh$, $r = 0, 1, 2, ..., n,\ h > 0$.

(ii) $f(x_0, x_1, ..., x_n) = \beta^{-n} F(t_0, t_1, ..., t_n)$ under the linear transformation $x = \alpha + \beta t$, where $x_i = \alpha + \beta t_i,\ i = 0, 1, ..., n$ and $f(\alpha + \beta t) = F(t)$.

(b) Describe the method of false position for finding a real root of an equation $f(x) = 0$ and obtain the corresponding iteration formula. Discuss its advantages and disadvantages in comparison to Newton-Raphson method. 7.5

8. (a) Explain the principle of numerical differentiation. Deduce Lagrange's numerical differentiation formula (without the error term). 7.5

(b) What do you mean by the open and closed type quadrature formula? Obtain trapezoidal rule for numerical integration without the error term. 7.5

9. (a) State Gauss' elimination method with pivoting for a system of linear equations $AX = B$, where $A = (a_{ij})_{m\times n}$, $X = (x_1, x_2, ... , x_n)^T$ and $B = (b_1, b_2, ... , b_m)^T$. 7.5

(b) Solve the equation $\frac{dy}{dx} = y^2 + yx,\ y(1) = 1$ by modified Euler's method to obtain $y(1.2)$ and $y(1.4)$. 7.5

10.(a) Discuss the bisection method for finding a simple real root of an equation $f(x) = 0$ lying in the interval $[a, b]$. Show that the method is certain to converge. 7.5

(b) Establish Newton's Backward interpolation formula. Where is this formula used? 7.5

SECTION-II

[Marks: 10]

Answer any *one* question from the following $10\times 1 = 10$

11.(a) (i) Draw the block diagram of a computer. 2+2+1

(ii) Define a bit and a byte.

(iii) What is a memory chip?

(b) (i) Convert $(A35)_{16}$ into binary. 2+2+1

(ii) Use 2's complement to compute $10100.01_2 - 11011.10_2$.

(iii) Find the CNF of $xy' + x'y$.

12.(a) Given the values of a, b, c the lengths of three segments. Write a FORTRAN 77/90 or C program to test whether they can form a triangle or not. 5

(b) Write a FORTRAN 77/90 or C program to find a real root of $xe^x + \log(1+x) - \sec\left(\sqrt{x^2+1}\right) = 0$ by the method of bisection. 5

13.(a) Write a FORTRAN 77/90 or C program to determine whether a number is prime or not. 5

(b) Write a FORTRAN 77/90 or C program to arrange the marks in Mathematics for 20 students in a class in descending order. 5

——×——